

Modeling and Uncertainty Quantification for Airfoil Icing

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Outline

1 Motivation/Background

2 Data-Based UQ

3 Computational UQ



Topic

1 Motivation/Background

2 Data-Based UQ

3 Computational UQ



Introduction

Wing icing deteriorates aerodynamics

- Leading edge flow separation bubble
- Lower lift, higher drag
- Unpredictable stall

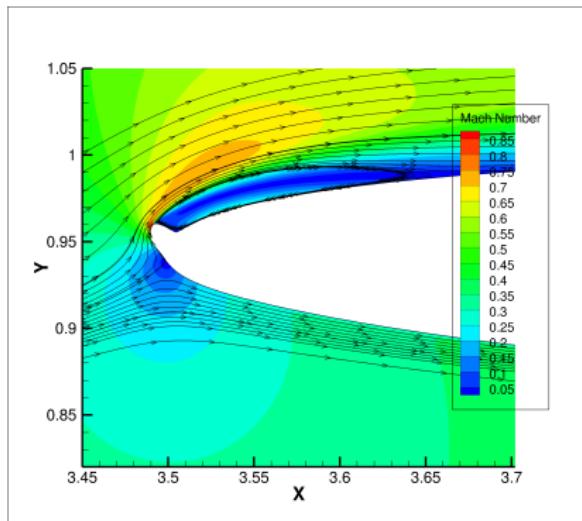


Figure: Leading edge horn separation



Introduction

Significant ice shape variation, sensitivity to physical parameters¹

- Complex physics (aero-thermodynamics, macro/micro scale physics)
- Uncertainty in physical parameters
- Nondeterministic variation (“same conditions, different shapes”)

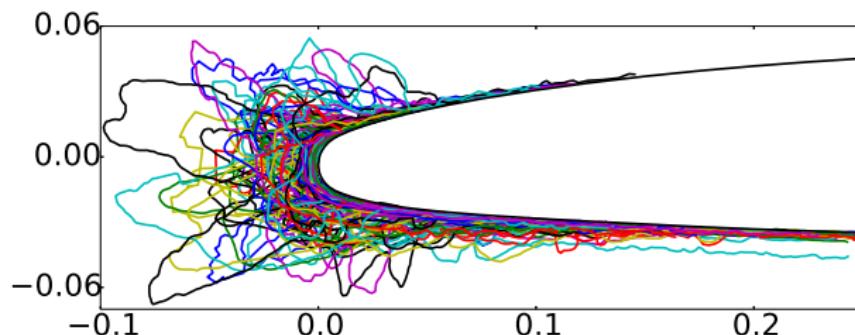


Figure: Wind tunnel experimental ice shapes

¹ Addy, H.E. *Ice Accretions and Icing Effects for Modern Airfoils*. NASA TR-2000-210031.

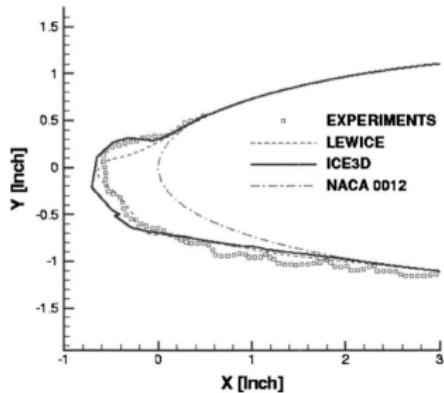




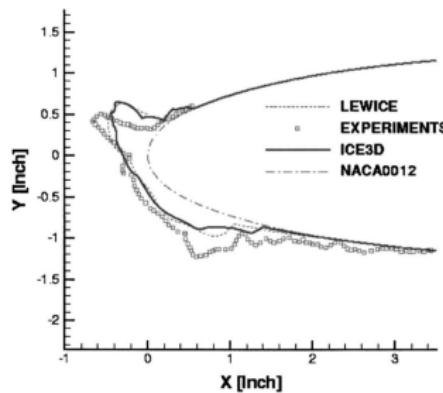
Introduction

Different types of ice accretion²

- “Rime” ice: cold temperature, low liquid water content
- “Horn” ice: warm temperature, high liquid water content
- Uncertainty in physical conditions can create uncertainty in performance



(a) Rime Ice



(b) Horn Ice

²Beaugendre et. al. *Development of a Second Generation In-Flight Simulation Code*. J. Fluids Engineering, 2006.



Introduction

Observations

- Many experimental ice accretion results exist, but no one has modeled this data or inferred anything from it
- Numerical methods/codes exist, but no one has systematically quantified statistical effects of parametric uncertainty in physics

Questions

- Can we infer/identify prominent ice shape features from database of shapes?
- Can we link physical information about data to ice shape features?
- Can we create a statistical modeling of icing using only data?
- Can we investigate parametric uncertainty in numerical icing codes?



Introduction

Research Goals

- Data-driven, equation-free model of ice accretion
 - Model ice accretion using *only* data, no physics
 - Study random ice shapes corresponding to a range of icing conditions
 - Could be used to benchmark/correct numerical calculations
 - Could be used to explore *non-deterministic* ice shape variations
- Computational model of ice accretion
 - Build numerical code to calculate ice accretion given physical inputs
 - Compute ice shape for a range of physical conditions
 - Perform parametric UQ and study effects on aerodynamic performance



Introduction

Research Approach

- Data-driven, equation-free modeling of ice accretion
 - Collect large number of ice shapes into a database
 - Model database shape features using Proper Orthogonal Decomposition (POD)
 - Link physical information to shape features
 - Build a purely data-driven, statistical ice accretion model (*no equations*)
 - Perform parametric UQ, study performance variations, etc.
- Computational model of ice accretion
 - Build droplet advection/impact simulator (C++)
 - Build icing thermodynamics simulator (C++)
 - Interface code with aerodynamic solver (FLO103)
 - Perform parametric UQ, study performance variations, etc.



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Dataset

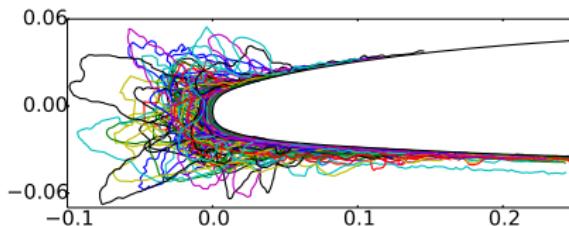


Figure: Wind tunnel experimental ice shapes

- Dataset consists of 145 experimental ice shapes
- Obtained in icing wind tunnel at NASA Glenn¹
- Representative of a wide variety of icing conditions (temperature, LWC, accretion time, etc.)



Data-Driven Model

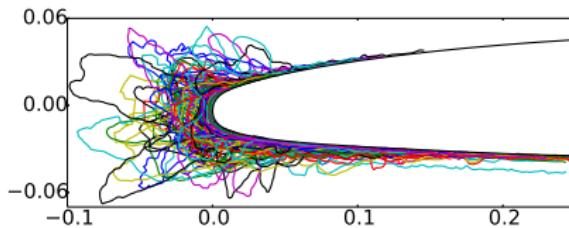


Figure: Wind tunnel experimental ice shapes

Goal: Make a purely data-driven model of icing (no equations)

Approach:

- Build low-dimensional model of shape using POD
- Correlate POD coefficients to temperature, accretion time, LWC
- Generate random ice shapes corresponding to given conditions



Proper Orthogonal Decomposition (POD)

Goal/Utility

- Statistical analysis tool
- Extracts linear, orthogonal basis that optimally explains dataset

Method

- Singular value decomposition of data matrix gives POD modes and eigenvalues

$$\mathbf{X} = \begin{bmatrix} | & & | \\ x_1 & \cdots & x_S \\ | & & | \end{bmatrix}$$

$$\mathbf{X} = \Phi \Sigma \mathbf{V}^*$$

$$x \approx \sum_i^M c_i \phi_i$$



POD Eigenvalues

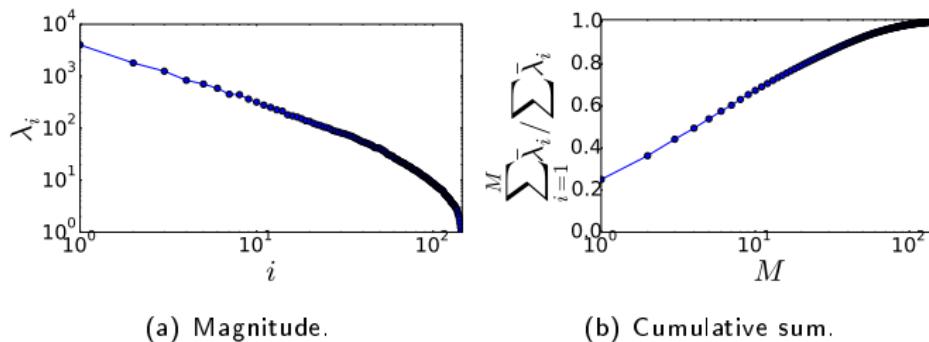


Figure: POD eigenvalues.

- 2/3 of cumulative sum contained in first 10 modes
- 2/3 of statistical variation contained in first 10 modes
- Truncate model at 10 modes



POD Modes

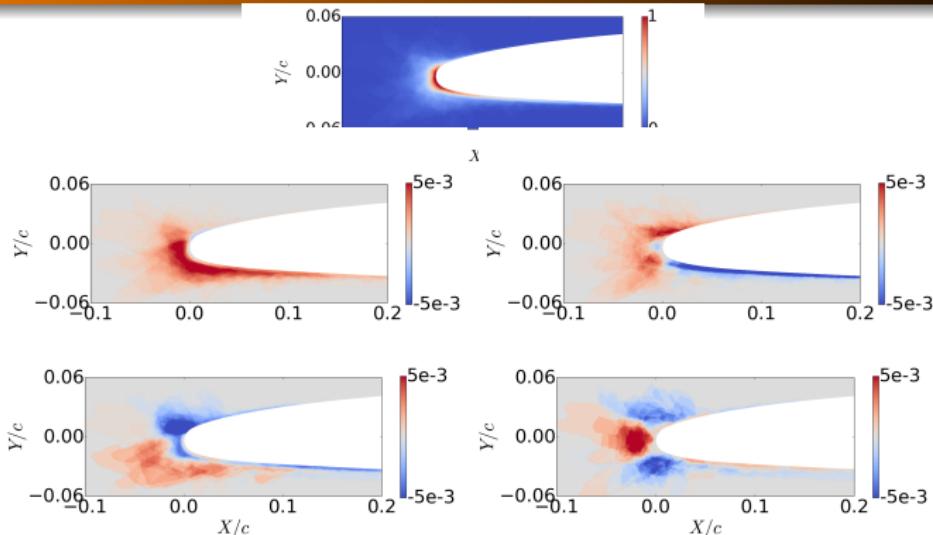


Figure: Mean and POD modes.

- Represent ice shapes as composite sum of these pictures
- Modes 1 and 2 simply add ice mass
- Modes 3 and 4 switch between upper/lower surface horns and rime
- Higher order modes contain more extreme shape perturbations



POD Reconstructions

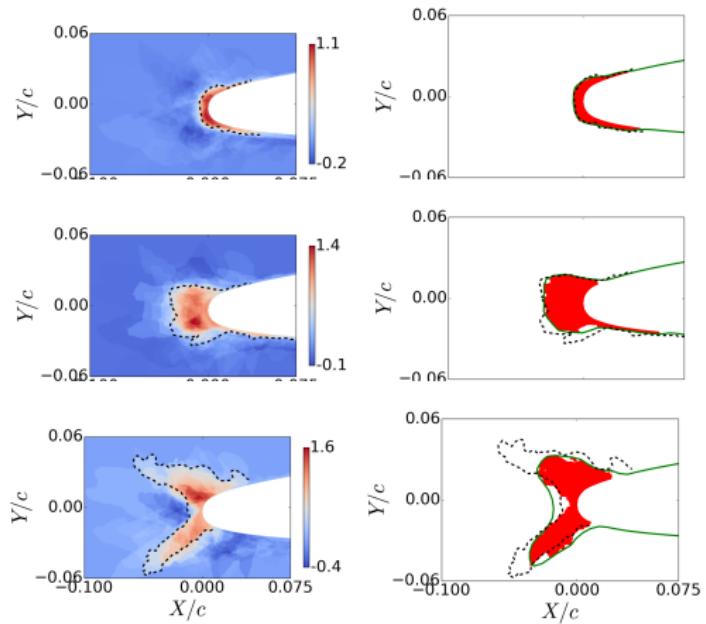


Figure: POD reconstructions.

- Agreement is great for shapes close to mean, less good for extreme shapes





Link Physical Conditions to Modes

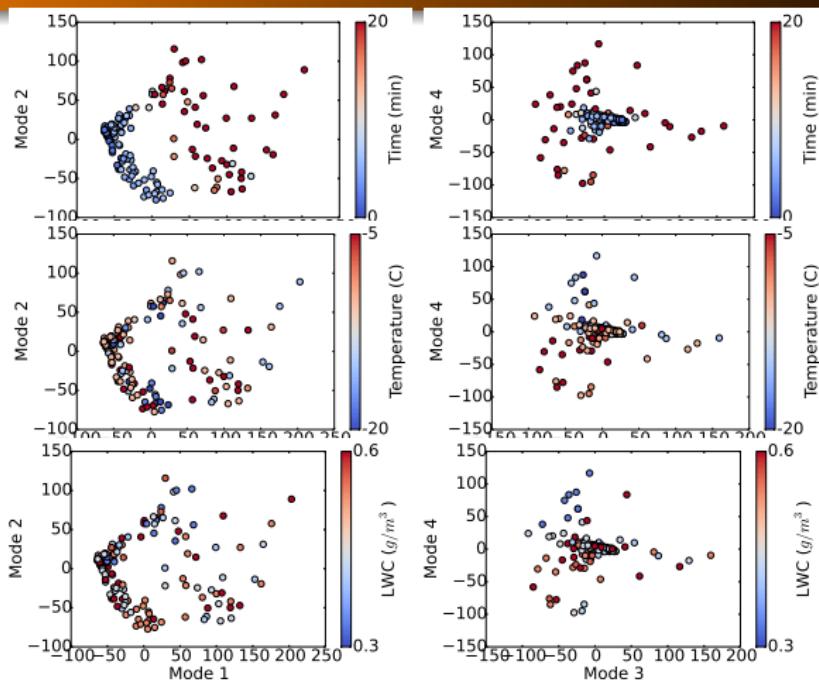


Figure: POD coefficients, colored with parameters.

- Statistically relate time, temperature and LWC to POD modes
- Input conditions, output POD coefficients that respect the data



Data-Driven Icing Model

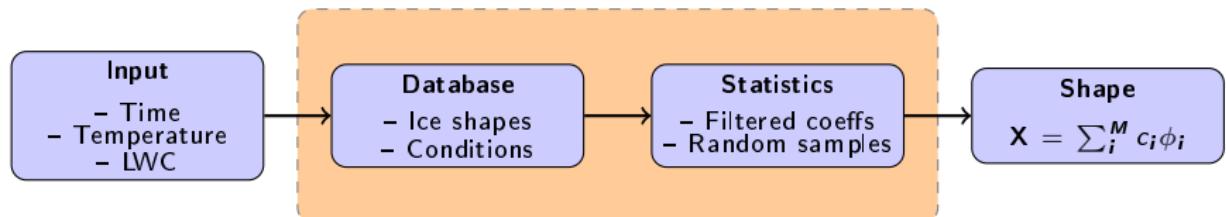
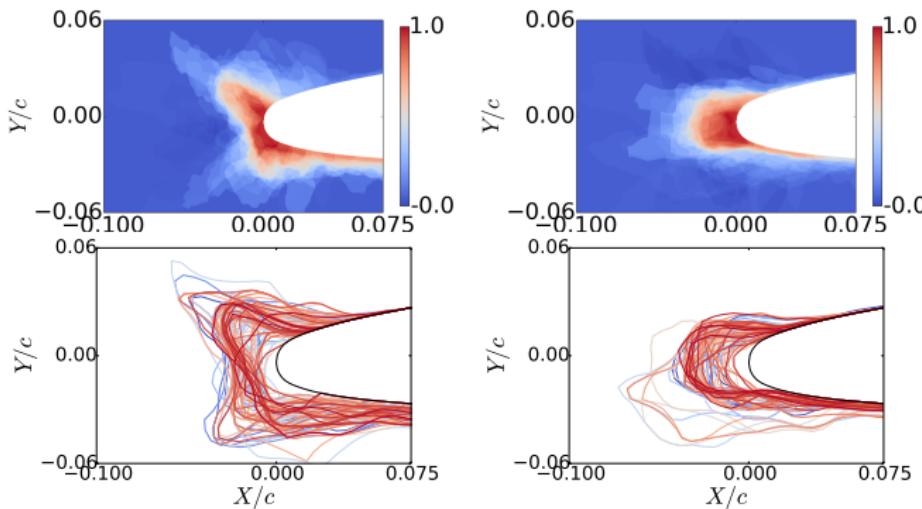


Figure: Flowchart of data-driven model.

- Input physical condition ranges
- Filter database for shapes that match conditions
- Create POD coefficient distributions for downselected data
- Generate random samples from these distributions
- Reconstruct ice shape using data-inferred POD coefficients



Random Shapes



(c) Time > 10 min; temperature $>$ 10 C; LWC $> 0.45 \text{ g/m}^3$

(d) Time > 10 min; temperature $<$ -10 C; LWC $< 0.45 \text{ g/m}^3$

Figure: Random data-driven ice shapes.

- These shapes were generated at random, no physics



Uncertainty Quantification

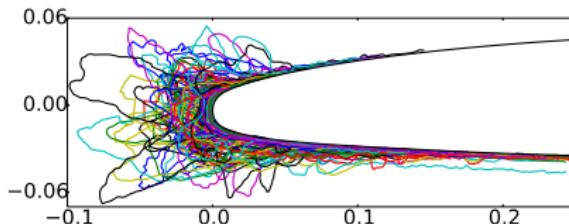


Figure: Wind tunnel experimental ice shapes

Goal: Quantify performance variation with POD modes

Approach:

- Generate random samples in POD space with Latin Hypercube Sampling (LHS)
- Test corresponding shapes with flow solver
- Quantify lift/drag statistics



Latin Hypercube Samples

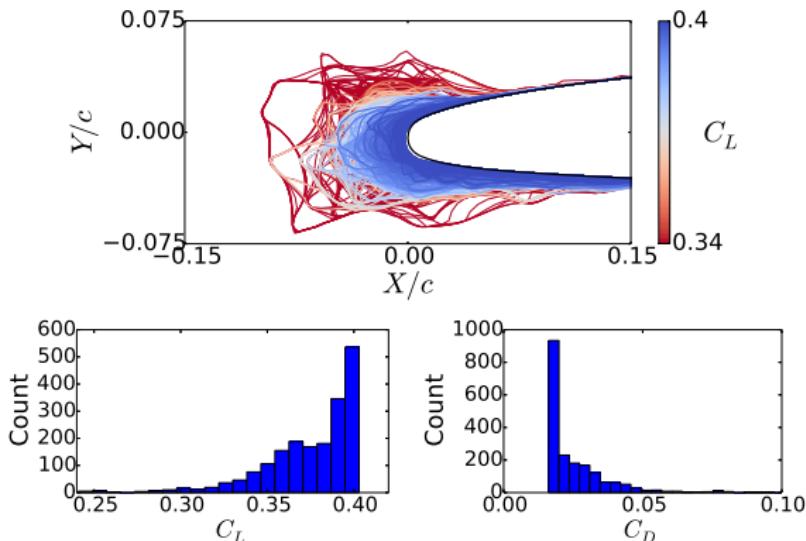


Figure: Latin Hypercube samples.

- 1,921 LHS samples from 10-D modal space
- LHS statistics reflect database statistics



Latin Hypercube Samples

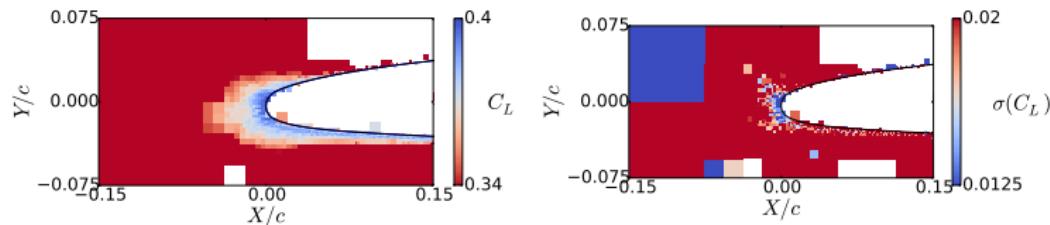


Figure: Spatial average and variance.

- Spatial average: lower surface rime accretions are relatively benign
- Spatial variance: performance sensitive to upper surface horns



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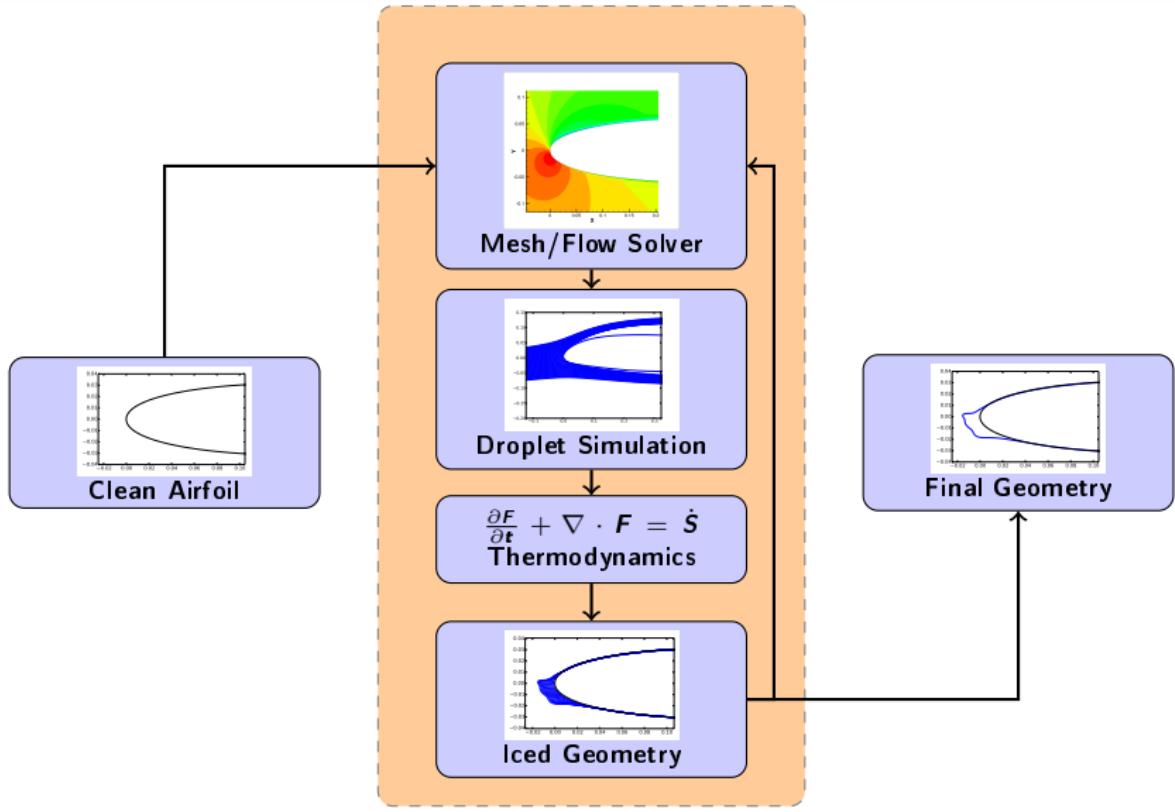
Motivation

Investigate uncertainty in the physical process of icing

- What is the statistical effect of uncertainty in physical parameters?
 - Free-stream temperature
 - Liquid water content (LWC)
 - Accretion time
 - Droplet diameter distribution (MVD)
 - Surface roughness
- Want to quantify how perturbations of the physics affects aerodynamics



Airfoil Icing Code Flowchart



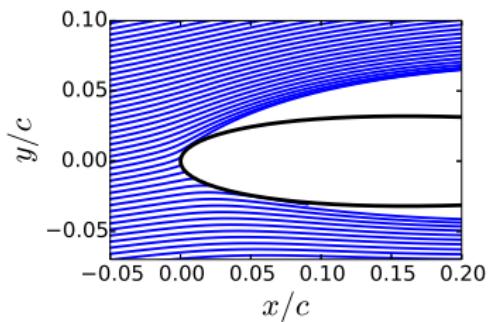
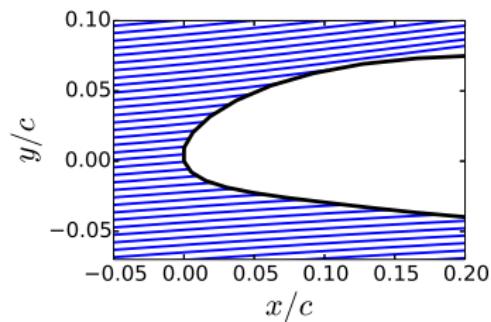


Droplet Advection

Advection Equations:

$$\frac{dx}{dt} = v$$

$$m \frac{dv}{dt} = \frac{1}{2} \rho_g C_D \pi r^2 ||v_g - v|| (v_g - v) + mg$$

**Figure:** $R = 10\mu\text{m}$ **Figure:** $R = 118\mu\text{m}$



Thermodynamics

Conservation Equations:

$$\begin{aligned}\rho_w \left\{ \frac{\partial h_f}{\partial t} + \nabla \cdot (\mathbf{u}_f h_f) \right\} &= \dot{m}_{imp} - \dot{m}_{evap} - \dot{m}_{ice} \\ \rho_w \left\{ \frac{\partial(h_f c_w T)}{\partial t} + \nabla \cdot (\mathbf{u}_f h_f c_w T) \right\} &= \left[c_w T_d + \frac{u_d^2}{2} \right] \dot{m}_{imp} \\ &\quad - L_{evap} \dot{m}_{evap} \\ &\quad + (L_{fus} + c_{ice} T) \dot{m}_{ice} \\ &\quad + c_H (T_{Rec} - T)\end{aligned}$$

- Mass
 - Enters through impinging droplets
 - Exits via evaporation/sublimation and freezing
- Energy
 - Enters through impinging droplets, freezing of ice
 - Exits via evaporation/sublimation, radiation, convection
- Solution procedure: explicit marching, finite volume discretization with upwinded derivatives



Thermodynamics Solution Procedure

$$\frac{\partial U}{\partial t} + \frac{\partial F}{\partial s} = \dot{S}$$

$$\int_{\mathcal{B}_k} \left(\frac{\partial U}{\partial t} + \frac{\partial F}{\partial s} \right) ds = \int_{\mathcal{B}_k} \dot{S} ds$$

$$\int_{\mathcal{B}_k} \frac{\partial U}{\partial t} \, ds + (F_{k+1/2} - F_{k-1/2}) = \int_{\mathcal{B}_k} \dot{S} \, ds$$

$$\frac{\partial \bar{U}_k}{\partial t} = \underbrace{\frac{1}{\Delta s_k} \int_{\mathcal{B}_k} \dot{S} \, ds - \Delta F_k}_{\delta_u}$$

$$\bar{U}_k^{t+\Delta t} = \bar{U}_k^t - \Delta t \delta_u$$

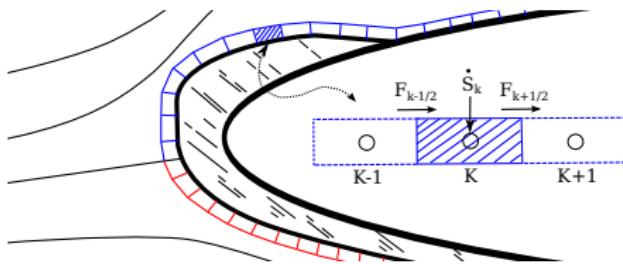


Figure: Finite volume method.

- Finite volume discretization of equations
 - Roe scheme flux calculation (upwinding)



Validations

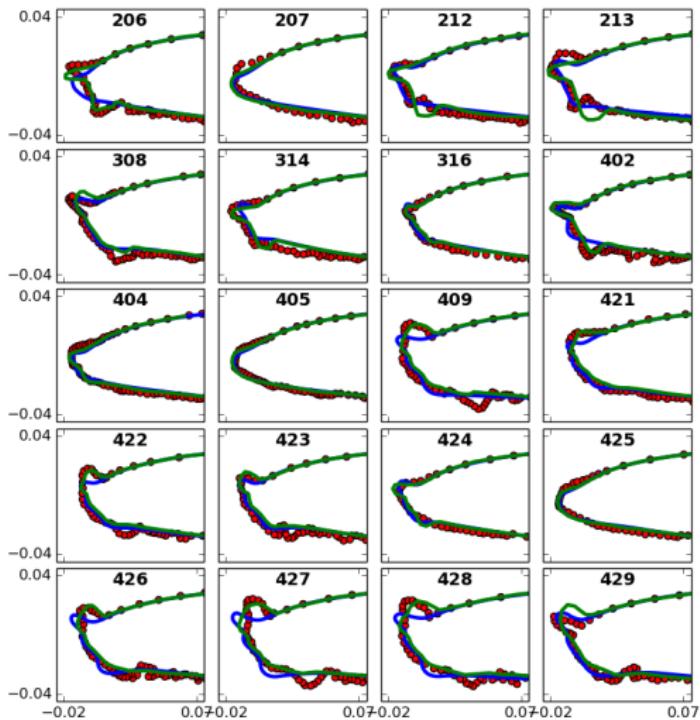
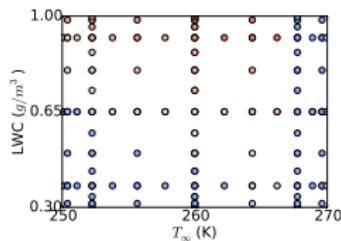


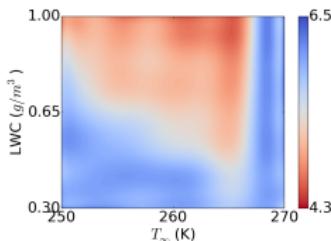
Figure: Experiment (red), LEWICE (green), and CATFISH (blue).



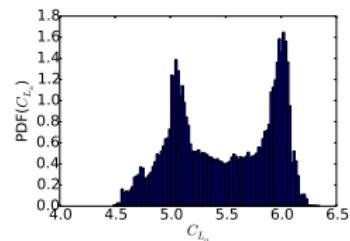
UQ Study: Temp + LWC



(a) Quadrature points (colors same as (b)).



(b) PCE surrogate.

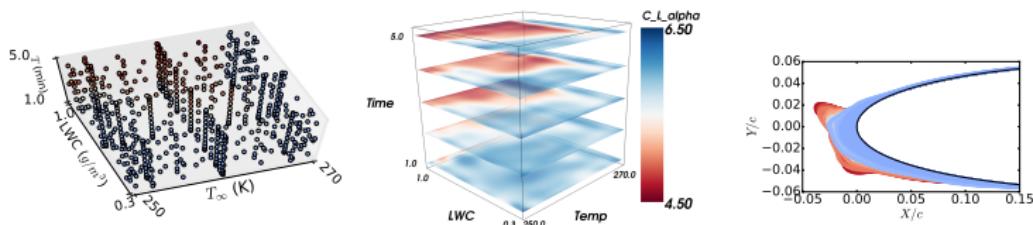


(c) PCE surrogate statistics.

Figure: Quadrature points, PCE surrogate, and statistics for the 2 parameter (T_∞ and LWC) study on lift slope.



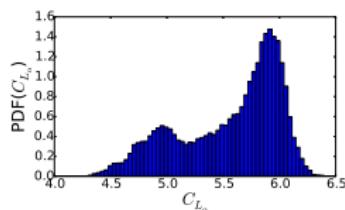
UQ Study: Temp + LWC + Time



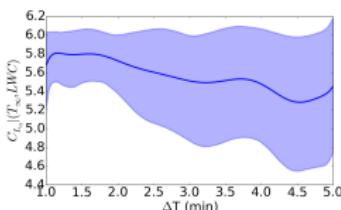
(a) Quadrature points (colorscale identical to (b)).

(b) PCE surrogate (parameter units identical to (a)).

(c) Ice shapes at quadrature points (colorscale identical to (c)).



(d) PCE surrogate statistics.

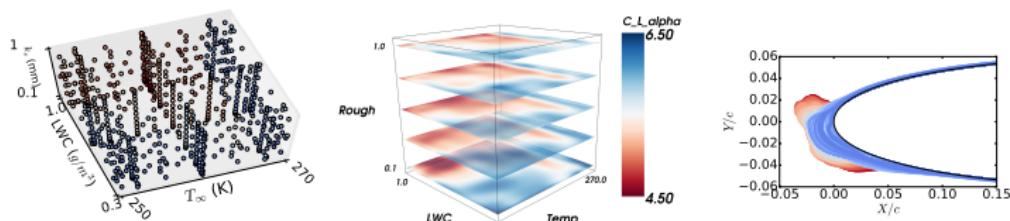


(e) Statistics of $C_{L\alpha}$ as a function of ΔT .

Figure: Quadrature points, PCE surrogate, and statistics.



UQ Study: Temp + LWC + Roughness



(a) Quadrature points (col-
orscale identical to (b)). (b) PCE surrogate (para-
meter units identical to (a)). (c) Ice shapes at quadra-
ture points (colorscape iden-
tical to (c)).

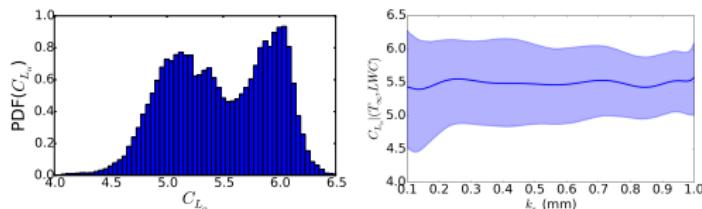


Figure: Quadrature points, PCE surrogate, and statistics.



Conclusions

Problems:

- Wing icing deteriorates icing aerodynamics, danger to safe flight
- Ice shapes are diverse and complex
- Not clear what the exact aerodynamic effects of different shapes are
- Would like systematic way of exploring airfoil icing through data

Solutions:

- Data-driven approach
 - Build empirical model of ice accretion from data
 - Perform parametric UQ to quantify range of performance
- Computational approach
 - Build computational ice accretion code from scratch
 - Perform parametric UQ to quantify effects of physics